

TESTING FOR WEEKLY SEASONAL UNIT ROOTS IN DAILY ELECTRICITY DEMAND: EVIDENCE FROM DEREGULATED MARKETS^{*}

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ABSTRACT

This paper analyses the nature of the weekly seasonal component in daily observations for the electricity demand series from several deregulated markets. We present and use the extension of the seasonal unit roots test of Hylleberg et al (1990) to the weekly seasonality case to formally determine whether the seasonal component of each variable exhibits stochastic non-stationarity. Daily demand series are taken from the Spanish, Argentine and Victoria State (Australia) Electricity Wholesale Markets. We find that only in the case of the Australian electricity demand there is evidence of unit roots, so the usual differentiating procedure employed in conventional time series models or regression approaches could imply a mis-specification.

Key Words: *Electric power, HEGY test, seasonal unit roots.*

JEL Classification: C12, C49, Q41

RESUMEN

Este trabajo examina la naturaleza del componente estacional semanal presente en las observaciones diarias de la demanda de electricidad de distintos mercados liberalizados. Se presenta y se utiliza la extensión del contraste de raíces unitarias estacionales de Hylleberg et al. (1990) para el caso de estacionalidad semanal en datos de frecuencia diaria, con la finalidad de determinar formalmente si el componente estacional de cada serie muestra o no comportamientos no-estacionarios. Las series de demanda se han obtenido de los mercados mayoristas de electricidad de Argentina, España y del estado de Victoria (Australia). La evidencia muestra que sólo en el caso de Victoria parecen existir raíces unitarias, por lo que el procedimiento habitual de diferenciación en la metodología de series temporales o de regresión implicaría la incorrecta especificación del auténtico proceso subyacente.

Palabras Clave: *Demanda eléctrica, contraste HEGY, raíces unitarias estacionales.*

Clasificación JEL: C12, C49, Q41

1. INTRODUCTION

Electricity industry has undergone a progressive deregulation process in an international context in the last decade. Among other important consequences liberalisation has driven electric power to be considered as another commodity more, traded under competitive principles on wholesale electricity markets. These markets are rapidly growing in trading and importance. Both daily electricity demand and daily spot prices quoted in power markets are characterised by a highly volatile behaviour that include strong seasonal patterns associated with different time intervals, such as the day of the week or the season of the year. The modelling and unbiased forecasting of future demand in short run are particularly interesting for market agents, since the non-storability nature of electricity makes it impossible to keep stocks to cover shocks in demand.

Time-series models are a common reference in a wide variety of empirical applications for modelling and forecasting economic and non-economic variables, because pure time series models could turn out as good as the econometric ones. Since the seminal work of Box and Jenkins (1970), investigators have often modelled seasonal time series using the ARIMA framework. This methodology is often based on the differentiation of series. Thus, the seasonal cycle s is usually removed from seasonal series by applying the differencing filter $(1 - B^s)$, where B is the back-shift operator. It is easy to show that this method implicitly imposes a unit root on the zero frequency (long run), as well as on all harmonics of the fundamental seasonal frequency¹ ($2\pi/s$) that could result in the over-differentiation of the true process. On the other hand, econometric approaches usually require deciding whether the dependent variable is covariance-stationary (i.e., weakly stationary) both in seasonal and non-seasonal series.

The objective of this paper is to analyse the nature of the weekly seasonal component of the daily electricity demand, since this type of data is likely to be integrated at the zero and some seasonal frequencies. This is due to the importance of the long run and the seasonal components in these variables. We analyse the demand series from some deregulated markets where electricity is traded under competitive principles, like those of Spain, Victoria State (Australia) and Argentina. The methodology applied is based on the seasonal unit roots testing procedure. The most widely used methodology to test for non-stationarity is the procedure developed by Hylleberg et al., (1990). This test was originally derived for quarterly seasonality and although it is currently extended for other seasonal cases, it is not extended

¹ The seasonal filter imposes s roots on the unit circle, implying that the spectral density of the time series is not bounded, neither on the long run nor in any of the seasonal frequencies. Thus, taking (seasonal) differences results in rather strong assumptions regarding the stochastic properties of the time series.

for weekly seasonality in data collected on a daily basis. This work also extends this test methodology to the weekly seasonality case and presents the relevant critical values under different specifications of the alternative hypotheses.

The use of this analysis in empirical applications is quite diverse. One of the key assumptions in the classic regression model is that the error term is serially uncorrelated. The repercussions that serial correlation might have on the estimation of the parameters depend on whether the series are covariance-stationary or not. It is well-known that the regression between integrated variables generates a significant relationship even when this is not really the case (the so-called spurious or false regression), so it is important to detect unit roots. Among seasonal integrated series there might also be long run cointegration relationships (Engle and Granger, 1987) and/or seasonal cointegration relationships (see Hylleberg et al., 1990; Lee, 1992; Johansen and Schaumborg, 1999; inter alia). De Vany and Walls (1999) analysed daily electricity spot prices from different state markets in the United States, finding evidence of cointegration relationships in the long run of these series. However, given the strong weekly seasonal behaviour of electricity spot price series, there might also be cointegration in weekly seasonal frequencies that could affect the results seen.

There is an important framework of literature concerned with empirical analyses of electricity demand covering many different objectives and using different methodologies, so that an exhaustive review of this topic is outside the scope of this paper. The methodologies used for short run electricity demand forecasting have mainly focused on ARIMA methodology, regression or exponential smoothing models (see Bunn and Falmer, 1985). More sophisticated methods like splines or nonparametric analysis have also been used (Harvey and Koopman, 1993; Rodríguez, 2000). The cointegration methodology have also been employed to outline long run relationships between the demand and other variables (Engle et al., 1985; Beenstock et al., 1999). In the empirical framework concerned with daily electricity demand in Spain, we should mention the work of Cancelo and Espasa (1996).

The paper is organised as follows. In Section 2, we describe the composition of data and the main characteristics of the time series analysed. We also describe the basic organisational features of electricity markets from which they come. In Section 3, seasonal unit roots test procedure is extended to cope with weekly seasonality and we apply it to verify the possible existence of stochastic trends in daily demand time series. Finally, the main conclusions are summarised in the fourth section.

2. DAILY SERIES OF ELECTRICITY DEMAND

The way in which the electric wholesale market or pool² and its regulatory system are organised differ among countries, since the characteristics of the electric industry previous to the deregulation process and its final objectives strongly condition them. Nevertheless, the prior deregulation process evolved in England and Wales has greatly influenced the subsequent liberalisation processes, so the electricity market organisation in most countries (as is the case of the market analysed in this paper) is similar to the market model applied in UK. The aim of the electric pool is to provide an effective infrastructure and a set of rules for the efficient wholesale trade of electricity between market participants under competitive principles. Although there are typically other important components, the electric pool is basically structured around an organised Spot Market managed by an operator (called a Market Operator) who co-ordinates the economical trade between suppliers (generators) and purchasers (retailers companies, end-use consumers and distributors). Generators compete among themselves in the Spot Market by submitting dispatch offers that include prices for different levels of their electric production, while purchasers submit dispatch bids comprising prices and quantities of the electric power that they will need³. The Market Operator aggregates all these dispatches and calculates the price and the quantity of electricity that balance supply and demand in constant intervals of time within the day. In the Spanish and in the Argentine markets, the time interval of reference is the hourly period. In those electricity markets more influenced by the England and Wales market model, as is the case of the Australian markets, the time interval of reference is the half hourly period⁴.

The series analysed in this work have been formed as the arithmetic average of the intra-daily series of quoted demand (measured in Gig-watts) from every wholesale electricity market. Thus, we have built two daily series through the averaging of the 24 hourly series of market-clearing demand for the Spanish and the Argentine wholesale electricity markets, and a third one by averaging the 48 half-hourly series of market-clearing demand for the National Wholesale Market of Victoria⁵. Data was collected in different sample periods from each market. We have data from 01/01/1998 to 31/10/2000 (1,035 observations) for the Spanish market; the period 01/05/1999 to 31/12/2000 (611 observations) was sampled for the Argentine market. Finally, we take data in the period 01/01/1997 to 28/02/2001 (1,520

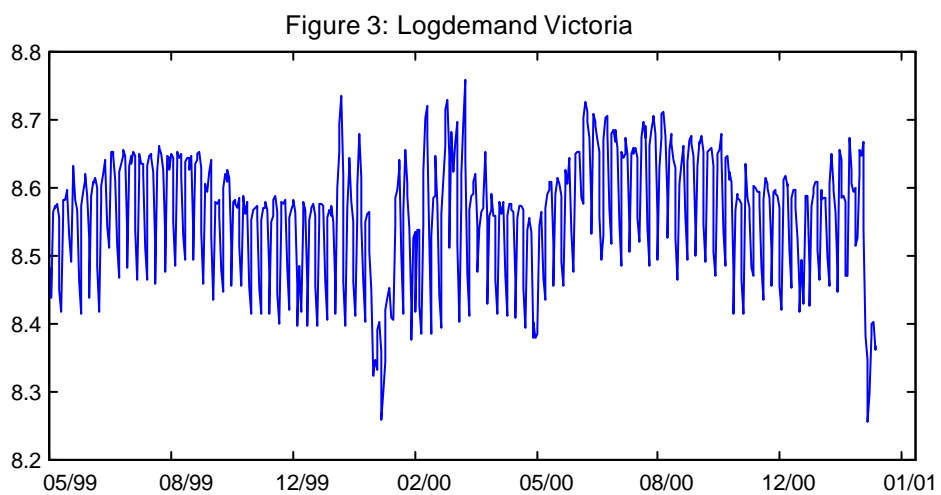
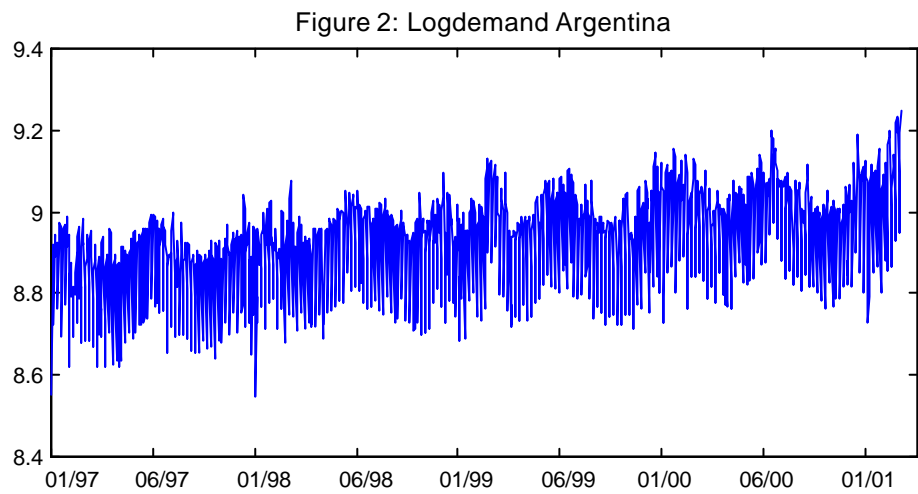
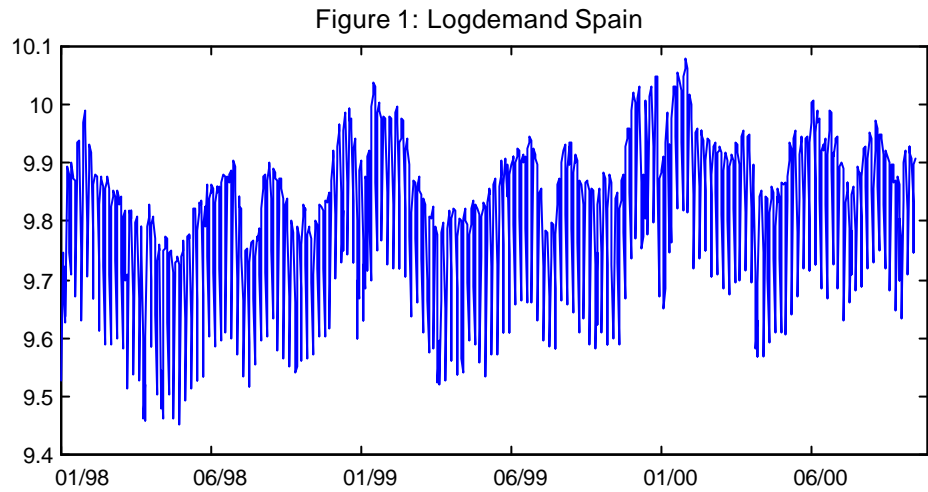
² Since electricity cannot be stored and it is not possible to distinguish which generator produced the electricity consumed by a particular customer, all the wholesale electricity markets involved use the concept of a pool, where all the generated electricity is centrally pooled and scheduled to meet the electricity demand.

³ The dispatches refer to the day following the current market session.

⁴ A more detailed survey of the specific characteristics or the market rules of every electricity market involved is outside the scope of this paper. A deeper survey can be found in López Milla (1999) for the Spanish Market; Mastrángelo (2001) for the Argentine Market and Wolak (1997) for the Australian market.

⁵ All data used in this paper are freely available in the official WebPages of respective Market Operators: www.omel.es (Spain), www.cammesa.ar (Argentina) and www.nemmco.au (Victoria).

observations) for the Argentine market. These series are then transformed by taking natural logarithms, which is the usual transformation that provides variance-stationarity as well as preserves the basic characteristics of the original time series. The graphics of the resulting time series are shown in Figures 1, 2 and 3.



The most important characteristics of the daily series for electricity demand are well-known, and they are essentially the same ones as those of daily spot prices. The most remarkable feature of daily electricity demand is its high volatility level, which can be related to factors that are originated by the economic activity and by variations in climate conditions over the year. Daily electricity demand also displays a strong seasonal component associated with weekly and annual periods. The weekly seasonality is fundamentally the result of the systematic decrease in electricity consumption during the weekends. In contrast, the annual seasonality is more related to the recurrent variation of the temperature during the different seasons of the year and to the economic activity level of the country during these seasons. Thus, in the Spanish market, for example, the demand increases significantly during the coldest months of winter and the hottest months of summer due to the intense use of heating and air-conditioning respectively. A clear relationship also exists between electricity demand and the economic level of the country. Thus, periods of continued growth in electricity consumption have been associated with economic cycles of expansion and hence greater productivity. In the case of Spain, we can clearly observe that the electricity demand shows a rising trend that can be explained by factors related to the economic growth observed during the second half of the nineties.

In general, daily average price series are highly correlated with daily demand series, both showing the same basic characteristics. However, electricity prices series often display features in their behaviour that are not present in demand dynamics. These differences are due to factors related to micro-structural characteristics of the electricity market, like the degree of technical development of the system, and factors related with the peculiarities of the electricity industry in the country, like the degree of horizontal concentration. In this sense, electricity prices sometimes show infrequent and dramatic jumps in the price level (commonly due to technical problems in the electric system) that, however, do not appear in the demand dynamics.

The autocorrelation function (ACF) of the log-demand time series (see Figures 4, 5 and 6) show the seasonal patterns quite clearly through significant correlations at the lags that are multiples of seven (weekly seasonality) and significant correlations around the lag 365 (annual seasonality). The correlograms also display a strong persistence at the level of the time series that could arise from deterministic or stochastic trends in the long run as well as in the weekly seasonal frequencies. The nature of the seasonal component of the series of log-demand is analysed in the following section through the seasonal unit roots testing procedure.

Figure 4: Correlogram Logdemand Spain

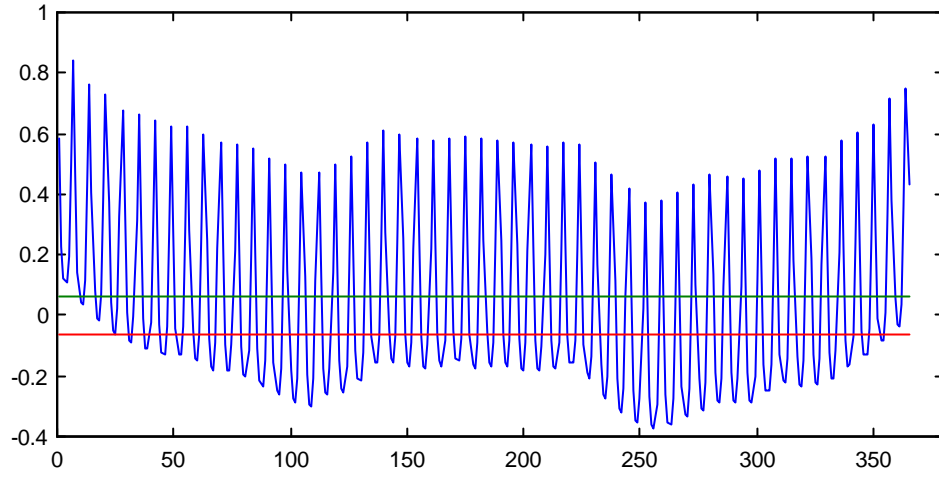


Figure 5: Correlogram Logdemand Argentina

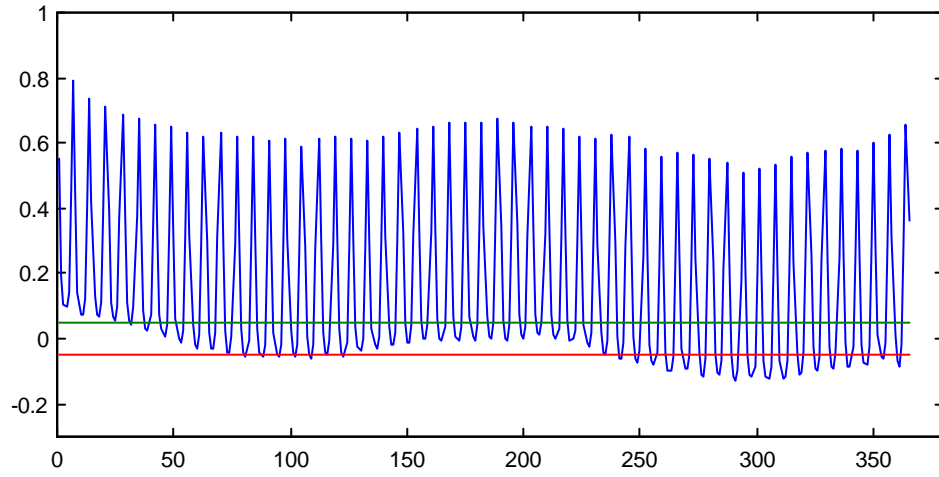
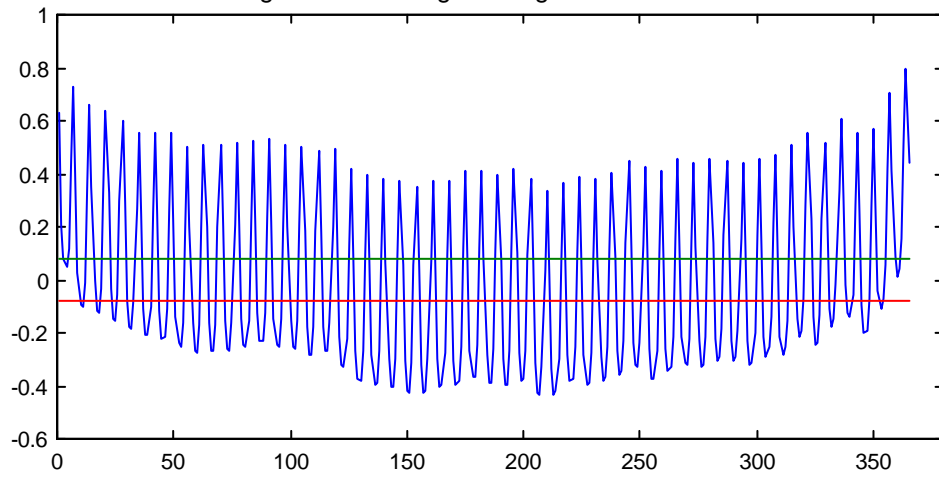


Figure 6: Correlogram Logdemand Victoria



3. TESTING FOR WEEKLY SEASONAL UNIT ROOTS

3.1. Theoretical considerations

The representation of a time series through a process that includes seasonal unit roots implies the existence of a time-varying seasonal structure over time, so the underlying generating process is non-stationary. On the other hand, the seasonality modelled through deterministic variables (like dummies or trigonometric regressors) assumes that the seasonal component is stable and fully predictable. Finally, stationary stochastic seasonality assumes that the process is stable, but allows a certain degree of random variation around a stationary mean. Obviously, modelling seasonality in some of these ways implies strong assumptions about the true data generating process (hereafter DGP), which could derive in important biases if these assumptions are not really verified (Ghysels and Perron, 1993).

The unit root test is a statistical tool that affords further information about which DGP would best fit the seasonal behaviour of the time series. Many methods have been developed to test non-stationarity in univariate time series, but the seasonal unit roots test most employed in empirical studies is the one of Hylleberg et al., (1990), (henceforth HEGY). Under the null hypothesis, the HEGY test assumes that the relevant variable is seasonally integrated. Thus, if we consider series of observations of daily frequency with a weekly seasonal component, the test procedure assumes that the series is determined through the DGP given by $(1 - B^7)y_t = \mathbf{e}_t \sim iid(0, \mathbf{S}_e^2)$; $t = 1, \dots, T$. The characteristic polynomial of this process can then be factorised as $(1 - B)S(B)$, where $S(B) = (1 + B + \dots + B^6)$ is the seasonal moving average filter. So, under the null hypothesis it is assumed that the variable has a single unit root at the zero frequency and three pairs of complex roots at the seasonal frequencies $k\mathbf{w}$; $k = 1, 2, 3$; $\mathbf{w} = 2\mathbf{p}/7$. Notice that k represents the number of cycles per week of each frequency. The seasonal unit roots testing procedure is based on the expansion of the characteristic polynomial about its roots⁶, determining the following auxiliary regression (see Appendix A for the technical aspects involved in the derivation of this expression):

$$\Delta_7 y_t = \mathbf{a} + \mathbf{b}t + \sum_{j=2}^7 \mathbf{a}_j D_{jt} + \sum_{j=2}^7 \mathbf{g}_j D_{jt}t + \sum_{j=1}^7 \mathbf{p}_j z_{j,t-1} + \sum_{r=1}^p \mathbf{f}_r \Delta_7 y_{t-r} + \mathbf{e}_t \quad (0.1)$$

$$\mathbf{e}_t \sim iid(0, \mathbf{S}_e^2)$$

where D_{jt} is a zero/one dummy corresponding to the j -th day of the week and each one of the regressors $z_{j,t}$ are defined on the seasonal frequencies $\{k\mathbf{w}; k = 1, 2, 3; \mathbf{w} = 2\mathbf{p}/7\}$ as follows:

⁶ See Hylleberg et al (1990), pp. 221-223.

$$\begin{aligned}
z_{1,t} &= \sum_{j=1}^7 \cos(0j) B^{j-1} y_t = S(B) y_t; \\
z_{2k,t} &= \sum_{j=1}^7 \cos(kj\omega) B^{j-1} y_t; \\
z_{2k+1,t} &= -\sum_{j=1}^7 \sin(kj\omega) B^{j-1} y_t;
\end{aligned} \tag{0.2}$$

The auxiliary regression (1.1) reflects the most general specification under the alternative hypothesis of stationarity. This specification includes a drift, a linear time trend, deterministic seasonal variables and seasonal drifts. The alternative specification of the test could include different combinations of these deterministic terms, either all, some or none of them (i.e., in a constrained version of the above model). The correct determination of the alternative hypothesis is important because the power of the test depends on this specification. Moreover, in a similar way to that of Dickey and Fuller (1979) unit root test procedure (the so-called ADF test), a number of lags of the dependent variable should be included in order to ensure serial uncorrelation in the error term. The choice of the number of lags p to be included in the regression supposes in practice a trade-off between the statistical properties of size and power. Unfortunately, there is no unanimous agreement on the best method to determine it (see Ng and Perron, 1995; and Psaradakis, 1995; for a discussion of different methods in order to apply the test).

The most remarkable feature of the HEGY testing procedure is that the series y_t is represented under the decomposition evolved under the null hypothesis as an orthogonal linear combination of the regressors $z_{j,t}, j = 1, \dots, 7$. Each one of these regressors is defined on a specific angular frequency by a linear filtering that removes all the unit roots from y_t except the one associated with the specific frequency of the regressor. Notice for example that $z_{1,t}$ is the result of filtering y_t with the seasonal moving average filter $S(B)$, so all the seasonal unit roots have been removed and $z_{1,t}$ can only include a unit root at the zero frequency. Furthermore, note that that unit root will only be present in this regressor because the filtering process removes it from all the other regressors.

Moreover, the polynomial expansion carried out determines that regressors are asymptotically mutually orthogonal (Beaulieu and Miron, 1993). This ensures that the result of testing the unit root hypothesis in a given frequency is not affected by the results of the testing procedure in the remaining frequencies. The important implication of this property is that the HEGY test allows us to determine whether a unit root exists in some, all or none of

the frequencies analysed, by means of the acceptance or the rejection of the significance of each of the parameters associated to each particular frequency.

By applying Ordinary Least Squares (OLS) to equation (0.1) or to any deterministic constrained version of it, we obtain the estimation of each one of the \mathbf{p}_j parameters. The series has a unit root on the zero frequency if the null hypothesis $\mathbf{p}_1 = 0$ is accepted with a t-statistic against a unilateral alternative hypothesis ($\mathbf{p}_1 < 0$). This test is completely similar to the augmented unit root test of Dickey and Fuller (1979) and, in fact, the distribution of the statistic is the same. The complex unit roots hypothesis on each seasonal frequency ($2\mathbf{p}_k/7; k=1,2,3$) implies that both statistics associated with the same seasonal frequency are zero ($\mathbf{p}_{2k} = \mathbf{p}_{2k+1} = 0$). This evidence can be tested either sequentially through the use of t-statistics (a two-sided test for \mathbf{p}_{2k+1} can be tested and if it is not rejected, then compute a one-sided test on \mathbf{p}_{2k}), or alternatively, a joint test can be applied. The second approach is normally preferred since it seems to have better statistical properties (Ghysels, Lee and Noh, 1994). The acceptance of the null implies that the series contains the complex roots e^{kwi} and e^{-kwi} . Neither the t-statistics nor the F-statistics have standard distributions and both depend, furthermore, on the set of deterministic variables included in the auxiliary regression. We determine the conventional critical values of those distributions by Monte Carlo simulation for different sample sizes and for different sets of deterministic regressors. The resulting critical values and a brief description of the simulation process employed are shown in Appendix B.

Finally, note that seasonal time series could be simply modelled by the seasonal differencing filter or through the use of a multiplicative differencing filter $(1-B)(1-B^s)$, since it is possible that the series exhibit more complicated dynamics. When time series are modelled within an ARIMA framework, both seasonal and non-seasonal differences are frequently taken (see for example, Cancelo and Espasa, 1990; for an application of this procedure to daily demand series). So, given the possibility of different specifications of non-stationary seasonality, one should select a testing strategy that is general enough to take into account all relevant cases. Moreover, it is well known that repeated roots in the same frequency influence the distribution of the relevant statistics (Dickey and Pantula, 1987; and Banarjee et al., 1993). Therefore, it is also important for statistical reasons to start at the highest order of integration and test downwards.

We can represent the order of integration of a seasonal series in a general sense by following the definition of Engle, Granger and Hallman (1988). These authors define a time variable y_t as *seasonally integrated of orders* d_0 and d_s , denoted as $SI(d_0, d_s)$, if the

process $(1-B)^{d_0} S(B)^{d_s} y_t$ is stationary and admits an invertible ARMA representation. So, we should follow a sequential testing procedure by firstly establishing the order of the parameters d_0 and d_s and test downwards the different hypothesis of non-stationarity. The statistical sequential procedure and some considerations about it are sketched in Appendix C.

In the case of daily demand series from electric markets, it is reasonable to consider that the maximum integration order could be represented as a $SI(2,1)$ process, because the growth (trend) over time seen in the series could be described appropriately through a stochastic trend⁷. In this case, a double unit root would exist in the level of the series and unit roots in all of the seasonal frequencies, so that it would be necessary to apply the multiplicative differencing filter to the original series to achieve stationarity.

3.2. Empirical results

We now present the results of applying the HEGY test procedure to the log-demand time series. In order to test the $SI(2,1)$ hypothesis, we must apply the HEGY test to the differenced series, that is, we have to impose a unit root on the level of the time series involved. Under the alternative of stationarity, the auxiliary regression could include a drift and seasonal dummies. The dummy variables allow different growth rates for the different seasons under the alternative. We do not include a linear time trend in the auxiliary regression because we are not yet testing against the trend stationary hypothesis. The seasonal unit roots test is then based on the following regression:

$$\Delta_7 \Delta_1 y_t = \mathbf{a}_0 + \sum_{j=2}^7 \mathbf{a}_j D_{jt} + \sum_{j=1}^7 \mathbf{p}_j z_{j,t-1}^* + \sum_{r=1}^p \mathbf{f}_r \Delta_7 \Delta_1 y_{t-r} + \mathbf{e}_t \quad (0.3)$$

where $z_{j,t}^* = \Delta_1 z_{j,t}$ and the remaining regressors have already been defined in the previous section.

Using the backward method commented in Ng and Perron (1995), additional lags are included in the regression in order to capture autocorrelation in the residuals. This procedure consists of selecting a maximum number of lags to be included and then removing those that fail to enter significantly at a certain level⁸. We tried different values of the lagged dependent

⁷ Empirical framework has shown that economical time series are typically $I(0)$ or $I(1)$, at most $I(2)$, where $I(j)$ denotes the order to integration at the zero frequency.

⁸ We chose the usual 5% level. The results below, however, are not qualitatively different from those obtained with a more restrictive 1%.

variable to be included in the auxiliary regression and finally selected 21 lags. Notice that this number is sufficiently high to completely remove all the low-order autocorrelations in residuals, although it cannot eliminate the correlations generated by the annual seasonality around the lag 365. In practice, the choice of p necessarily supposes a trade-off between the statistical properties of the unit root test. On the one hand, if this number is too low and thus the correlation is not removed, the test size increases. On the other hand, if p is too large, the test power declines. It should be clear that the relevant distortion on statistical test properties really arises from residual correlation in the low-order lags, so they should be removed by including the number of lags that are necessary, usually rather low. In contrast, the residual correlation in high-level lags implies moderate shocks on the test size, whereas the inclusion of a massive number of lagged terms in the auxiliary regression in order to remove it generates drastic losses in the test power. Thus, the residual correlation due to annual seasonality could be removed by including 365 lags of the dependent variable in the auxiliary regression. However, the inclusion of such a large number of variables, most of which are irrelevant, it would reduce the test power practically to zero. Therefore, the contrast loses almost totally its ability to reject the null hypothesis, independently of the alternative hypothesis specified. Definitively, the effect of the high level-order lag residual correlation due to annual seasonality on the test size is expected to be insignificant compared to the distortion on power test caused when a large number of lags are included in order to remove it.

We test the serial uncorrelation hypothesis in the regression residuals by means of the usual Ljung-Box portmanteau test. The main results of the HEGY test are shown in Table 1. The evidence strongly indicates that in all the cases the first difference of the log time series is stationary. We find this not only in the level but also in all the seasonal frequencies, so we can conclude at the usual degrees of confidence that the data generating process $SI(2,1)$ hypothesis should be rejected.

Since the first difference of a stationary process is also a stationary process, we should now verify that the imposition of the unit root on the zero frequency was adequate. This evidence is obtained by applying the HEGY test to the level of the time series. The alternative now includes all the deterministic regressors, since the loss of power that results from their inclusion when unnecessary is insignificant compared to the bias that results from their omission when it is necessary (Beaulieu and Miron, 1993; Ghysels, Lee and Noh, 1994).

Table 1: HEGY Statistics: SI(2,1) process

HEGY statistics of the weekly seasonal unit roots test for the first difference of the log-demand series. The auxiliary regression includes a drift and seasonal dummies. The column “Lags” indicates the number of lags of the dependent variable included in the auxiliary regression.

The last row shows the statistics of the Ljung-Box portmanteau test of serial uncorrelation at the first 190 correlations in the regression residuals (p-value in brackets).

Significance in brackets: 1% (***), 5% (**), 10% (*).

Frequency	Statistic	Spain	Argentina	Victoria
0	\mathbf{p}_1	-7.75(***)	-10.56(***)	-6.61(***)
$2\mathbf{p} / 7$	\mathbf{p}_2	-7.52(***)	-9.47(***)	-5.72(***)
	\mathbf{p}_3	-2.67(**)	-3.26(***)	-1.05
$4\mathbf{p} / 7$	\mathbf{p}_4	-7.52(***)	-8.24(***)	-7.35(***)
	\mathbf{p}_5	-0.97	-1.05	-0.31
$6\mathbf{p} / 7$	\mathbf{p}_6	-8.71(***)	-10.27(***)	-6.35(***)
	\mathbf{p}_7	0.25	2.40(**)	-2.11(*)
$2\mathbf{p} / 7$	$F_{2,3}$	32.29(***)	50.77(***)	17.00(***)
$4\mathbf{p} / 7$	$F_{4,5}$	28.81(***)	34.55(***)	27.16(***)
$6\mathbf{p} / 7$	$F_{6,7}$	37.98(***)	56.03(***)	23.93(***)
Lags		21	21	17
Residuals	Q(190)	210.95(0.14)	168.68(0.86)	194.51(0.39)

So the test is now based on the auxiliary regression given by (0.1). We can trivially rewrite this expression and test the unit root hypothesis by means of the following auxiliary regression:

$$\Delta_7 y_t = \sum_{j=1}^7 \mathbf{a}_j^* D_{jt} + \sum_{j=1}^7 \mathbf{g}_j^* D_{jt} t + \sum_{j=1}^7 \mathbf{p}_j z_{j,t-1} + \sum_{r=1}^p \mathbf{f}_r \Delta_7 y_{t-r} + \mathbf{e}_t \quad (0.4)$$

$$\mathbf{e}_t \sim iid(0, \mathbf{S}_e^2)$$

Note that the equivalent specifications (0.1) and (0.4) permit the drift in a seasonal random walk DGP to differ across seasons, thereby allowing the amplitude of the variations across the seasons of the deterministic component in the level of the time series to vary

linearly over time. One further implication of this formulation is that it allows one to test the unit root hypothesis against the alternative of trend stationarity not only at the zero frequency, but also at the seasonal frequencies. The original model of Hyllebert et al., (1990) consider regressions which may only include intercept, seasonal intercepts and a trend variable. Afterwards, Smith and Taylor (1998) and Taylor (1998) generalised this test for a more flexible specification by including seasonal drifts in quarterly and monthly scenarios respectively. The number of lags of the dependent variable used are the same as those in the early phase of the testing procedure, since there are no relevant changes in this number when we freely apply the same criterion as employed above. In Table 2.1, the outcome of the seasonal unit roots test is shown.

Table 2.1: HEGY Statistics: SI(1,1) Process

HEGY statistics of the weekly seasonal unit roots test for the log-demand series. The auxiliary regression includes a drift, seasonal dummies, a linear time trend and seasonal drifts. The column “Lags” indicates the number of lags of the dependent variable included in the auxiliary regression.

The last row shows the statistics of the Ljung-Box portmanteau test of serial uncorrelation at the first 190 correlations in the regression residuals (p-value in brackets).

Significance in brackets: 1% (* * *), 5% (* *), 10% (*).

Frequency	Statistic	Spain	Argentina	Victoria
0	p_1	-4.48(***)	-4.62(***)	-2.46
$2p / 7$	p_2	-6.40(***)	-7.36(***)	-3.35
	p_3	5.24(***)	6.72(***)	4.74(***)
$4p / 7$	p_4	-6.48(***)	-7.38(***)	-6.07(***)
	p_5	3.57(***)	4.21(***)	4.20(***)
$6p / 7$	p_6	-8.26(***)	-9.29(***)	-6.78(***)
	p_7	1.97	4.58(***)	-0.54
$2p / 7$	$F_{2,3}$	35.33(***)	51.31(***)	17.31(***)
$4p / 7$	$F_{4,5}$	27.98(***)	36.78(***)	27.62(***)
$6p / 7$	$F_{6,7}$	36.26(***)	54.98(***)	23.58(***)
Lags		21	21	17
Residuals	Q(190)	224.14(0.04)	180.68(0.67)	193.25(0.42)

The results seem to show different evidence from time series. In the case of the demand series from the Spanish and Argentine markets, the presence of unit roots is rejected in all frequencies. The test also shows that both series are trend stationary and contain significant seasonal drifts in the weekly seasonal component. On the other hand, the unit root hypothesis at the zero frequency cannot be rejected in the case of the Australian log-demand, so the best specification in this case would be the $SI(1,0)$ process. It is interesting to remark that these results do not seem to change qualitatively when we do not allow seasonal drifts in the auxiliary regression, i.e., when model (0.1) is applied including only a common linear trend (i.e., $\mathbf{g}_j = 0$; $j = 2, \dots, 7$). The main results from this test are presented in Table 2.2. Given the observed features of data, auxiliary regressions based on a more restricted specification (i.e., without including linear trend or seasonal dummies) do not seem to be appropriate.

Table 2.2: HEGY Statistics: SI(1,1) process with linear trend

HEGY statistics of the weekly seasonal unit roots test for the log-demand series. The auxiliary regression includes a drift, seasonal dummies and a linear time. The column "Lags" indicates the number of lags of the dependent variable included in the auxiliary regression.

The last row shows the statistics of the Ljung-Box portmanteau test of serial uncorrelation at the first 190 correlations in the regression residuals (p-value in brackets).

Significance in brackets: 1% (* * *), 5% (* *), 10% (*).

Frequency	Statistic	Spain	Argentina	Victoria
0	\mathbf{p}_1	-4.44(***)	-4.59(***)	-2.78
$2\mathbf{p} / 7$	\mathbf{p}_2	-5.92 (***)	-7.22(***)	-3.27
	\mathbf{p}_3	4.94(***)	6.59(***)	5.38(***)
$4\mathbf{p} / 7$	\mathbf{p}_4	-6.26(***)	-6.92(***)	-6.09(***)
	\mathbf{p}_5	3.50(***)	4.00(***)	3.60(***)
$6\mathbf{p} / 7$	\mathbf{p}_6	-8.17(***)	-9.17(***)	-6.64(***)
	\mathbf{p}_7	1.98	4.56(***)	0.00
$2\mathbf{p} / 7$	$F_{2,3}$	30.67(***)	49.52(***)	20.15(***)
$4\mathbf{p} / 7$	$F_{4,5}$	26.29(***)	32.51(***)	22.25(***)
$6\mathbf{p} / 7$	$F_{6,7}$	35.58(***)	53.74(***)	22.28(***)
Lags		21	21	17
Residuals	Q(190)	216.00(0.19)	177.01(074)	188.30(0.52)

4. CONCLUDING REMARKS

The objective of this paper was, firstly, to present the extension of the seasonal unit roots test developed by Hylleberg et al., (1990) to the case of weekly seasonality and secondly, to apply it to the series of daily electricity demand quoted in several deregulated electricity markets. This methodology could be useful in determining the nature of the data generating process underlying daily electricity time series in empirical applications in which formal statements about the stationarity of these series are necessary, such as cointegration, regression and standard time series analysis.

The evidence obtained from applying the HEGY test procedure to electricity log-demand time series shows different seasonal behaviour patterns in the electricity markets, which are not necessarily non-stationary in the short samples considered in this paper. Therefore, the general conclusion would be that the mechanical application of the ARIMA methodology on these series, which fails to incorporate deterministic variables like seasonal dummies or seasonal trends, will imply the mis-specification of the true DGP, thereby introducing unit roots into the moving average component of the time series. These results should be taken with due caution owing to the relatively small size of the sample periods considered here, since they are limited by the recent creation of the electricity markets. As such, it is very likely that the daily demand time series display non-stationary dynamics when sample periods covering a larger number of years would be considered.

There is no unanimous agreement on the effect that the over-differentiation of time series has on the quality of forecasting. Moreover, there is very little literature on the effects of imposing seasonal unit roots on stationary processes. Diebold and Kilian (2000) have experimentally shown that the smaller the sample size and the further the prediction horizon, the greater the mean-squared forecast errors (MSE) in stationary processes with an imposed unit root will be. With shorter time horizons and larger samples, however, the reduction in the regression error due to the imposition of the unit roots (thus estimating a constrained model) could result in forecasts with the smallest MSE. We may consider this result extendible to the case of seasonal series, in which what it is considered, is the possibility that the process is a seasonal random walk. In this sense, O'Connor and Kapoor (1984) compare the results of ARIMA modelling with those obtained from models with seasonal deterministic variables in electricity demand series. While the forecasts of ARIMA framework show a better behaviour in the out-of-sample context, the alternative models seem to describe the dynamics of the series better in the in-sample context.

The real importance of unit root test is that it allows us to obtain a higher knowledge about the data generating process of the sampled series, thereby determining the parsimonious model that would best describe the behaviour of these series. The final decision on whether differentiating the process could depend on the purpose pursued by the investigator and the elements that surround the analysis, but there is no doubt that the evidence obtained through this tool provides more objective information to help the making of such a decision.

APPENDIX A: HEGY WEEKLY SEASONAL UNIT ROOTS TEST

Let the following data generating process (DGP) be:

$$(1-B^7)y_t = \mathbf{e}_t; \quad \mathbf{e}_t \sim iid(0, \mathbf{S}_e^2) \quad (\text{A.1})$$

Let $\mathbf{w} = 2\mathbf{p}/7$ be the fundamental weekly seasonal frequency and let $I \equiv \{1, 2, 3\}$ a set of subscripts. We can expand the polinomial $1-B^7$ about its roots⁹ as follows:

$$\begin{aligned} 1-B^7 = & \mathbf{I}_1 S(B) + \sum_{k \in I} \mathbf{I}_{2k} (e^{-k\mathbf{w}i} - B)(1-B) B \prod_{\substack{j \in I \\ j \neq k}} (1 - 2\cos j\mathbf{w} B + B^2) \\ & + \sum_{k \in I} \mathbf{I}_{2k+1} (e^{k\mathbf{w}i} - B)(1-B) B \prod_{\substack{j \in I \\ j \neq k}} (1 - 2\cos j\mathbf{w} B + B^2) \\ & + \mathbf{j}^*(B)(1-B^7) \end{aligned} \quad (\text{A.2})$$

where $S(B) \equiv \sum_{j=1}^7 B^{j-1}$ is the seasonal moving average filter and $\mathbf{j}^*(B)$ is a polinomial on B (the back-shift operator) with all its roots outside of the unit circle. Since $1-B^7$ is a real polinomial, each one of the complex pairs of constants $\{\mathbf{I}_{2k}, \mathbf{I}_{2k+1} : k \in I\}$ related to the complex roots, must be complex conjugated. The constant \mathbf{I}_1 associated with the only real root, is thus a real value. We can implicitly define the real parameters \mathbf{p}_j with $j = 1, \dots, 7$ as:

$$\mathbf{I}_1 = -\mathbf{p}_1, \quad \mathbf{I}_{2k} = \frac{1}{2}(-\mathbf{p}_{2k} + \mathbf{p}_{2k+1}i), \quad \mathbf{I}_{2k+1} = \frac{1}{2}(-\mathbf{p}_{2k} - \mathbf{p}_{2k+1}i); \quad k \in I \quad (\text{A.3})$$

By substituting the \mathbf{p}_j values of (A.2) for the \mathbf{I}_j parameters of (A.3) and simplifying gives:

$$\begin{aligned} 1-B^7 = & -\mathbf{p}_1 S(B) B - \sum_{k \in I} \mathbf{p}_{2k} (\cos(k\mathbf{w}) - B)(1-B) B \prod_{\substack{j \in I \\ j \neq k}} (1 - 2\cos(j\mathbf{w}) B + B^2) \\ & + \sum_{k \in I} \mathbf{p}_{2k+1} \sin(k\mathbf{w})(1-B) B \prod_{\substack{j \in I \\ j \neq k}} (1 - 2\cos(j\mathbf{w}) B + B^2) + \mathbf{j}^*(B)(1-B^7) \end{aligned} \quad (\text{A.4})$$

⁹ See Hylleberg et al (1990), pp 221-223.

The polynomials in the above expression that are multiplying the $\mathbf{p}_{2k}B$ and $\mathbf{p}_{2k+1}B$ terms can be simplified in the following way:

$$(\cos(k\mathbf{w}) - B)(1 - B) \prod_{\substack{j \in I \\ j \neq k}} (1 - 2\cos(j\mathbf{w})B + B^2) = \sum_{j=1}^7 \mathbf{x}_j B^{j-1} \quad (\text{A.5})$$

$$\sin(k\mathbf{w})(1 - B) \prod_{\substack{j \in I \\ j \neq k}} (1 - 2\cos(j\mathbf{w})B + B^2) = \sum_{j=1}^7 \mathbf{x}_j^* B^{j-1} \quad (\text{A.6})$$

where¹⁰

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{x}_6 = \cos(k\mathbf{w}) & ; & \quad \mathbf{x}_1^* = -\mathbf{x}_6^* = \sin(k\mathbf{w}) \\ \mathbf{x}_2 &= \mathbf{x}_5 = \cos(2k\mathbf{w}) & ; & \quad \mathbf{x}_2^* = -\mathbf{x}_5^* = \sin(2k\mathbf{w}) \\ \mathbf{x}_3 &= \mathbf{x}_4 = \cos(3k\mathbf{w}) & ; & \quad \mathbf{x}_3^* = -\mathbf{x}_4^* = \sin(3k\mathbf{w}) \\ \mathbf{x}_7 &= 1 & ; & \quad \mathbf{x}_7^* = 0 \end{aligned}$$

Thus we can express (A.5) and (A.6) more conveniently as:

$$B^6 + \cos(k\mathbf{w})[1 + B^5] + \cos(2k\mathbf{w})[B + B^4] + \cos(3k\mathbf{w})[B^2 + B^3] = \sum_{j=1}^7 \cos(jk\mathbf{w})B^{j-1}$$

$$\sin(k\mathbf{w})[1 - B^5] + \sin(2k\mathbf{w})[B - B^4] + \sin(3k\mathbf{w})[B^2 - B^3] = \sum_{j=1}^7 \sin(jk\mathbf{w})B^{j-1}$$

where the right side of the above equalities holds because of the cyclical properties of the trigonometrical functions involved. We finally express equation (A.4) as:

$$\begin{aligned} 1 - B^7 &= -\mathbf{p}_1 B \sum_{j=1}^7 B^{j-1} - \sum_{k \in I} \mathbf{p}_{2k} B \sum_{j=1}^7 \cos(jk\mathbf{w})B^{j-1} \\ &\quad + \sum_{k \in I} \mathbf{p}_{2k+1} B \sum_{j=1}^7 \sin(jk\mathbf{w})B^{j-1} + \mathbf{j}^*(B)(1 - B^7) \end{aligned}$$

¹⁰ The procedure for obtaining the coefficients $\mathbf{x}_j, \mathbf{x}_j^*$ is based on a rather tedious algebraic derivation, which is centred in some specific and general properties of the angular frequencies involved. The full proof is thus omitted, but it is available from the author upon request.

If we consider both the DGP in (A.1) and the above equation, we can write the basic auxiliary regression in the HEGY methodology (by assuming zero mean) as follows:

$$\begin{aligned} \mathbf{j}^*(B)\Delta_7 y_t = & \mathbf{p}_1 \sum_{j=1}^7 y_{t-j} + \sum_{k \in I} \mathbf{p}_{2k} \sum_{j=1}^7 \cos(jk\omega) y_{t-j} \\ & - \sum_{k \in I} \mathbf{p}_{2k+1} \sum_{j=1}^7 \sin(jk\omega) y_{t-j} + \mathbf{e}_t \end{aligned}$$

Finally, by adding a non-zero deterministic mean that includes different terms (such as a drift, seasonal dummies, seasonal and non-seasonal linear trends) and taking into account that $\mathbf{j}^*(B)\Delta_7 y_t$ represents a stationary autoregressive $\text{AR}(p)$ process, $(\mathbf{j}^*(B) \equiv 1 - \sum_{r=1}^p \mathbf{f}_r B^r)$, we can generalise the HEGY auxiliary regression to the less restrictive context of the main text.

APPENDIX B: CRITICAL REGIONS OF THE HEGY WEEKLY SEASONAL UNIT ROOTS TEST.

We represent as t_j the t-statistics of the individual test $\mathbf{p}_j = 0$ ($j=1, \dots, 7$) and we denote as $F_{2k, 2k+1}$ the F-statistic of the joint test $\mathbf{p}_{2k} = \mathbf{p}_{2k+1} = 0$, $k=1, 2, 3$.

We simulated 24,000 times the data generating process $\Delta_7 y_t = \mathbf{e}_t$, $\mathbf{e}_t \sim N(0, 1)$ and the usual critical regions were then computed from the auxiliary regression $\Delta_7 y_t = \mathbf{m}_t + \sum_{j=1,7} \mathbf{p}_j z_{j,t} + \mathbf{e}_t$ for the relevant statistics. We used different finite sample sizes ($T=240, 480$ y $1,000$) to define the temporal horizon of those regressions, taking $T=1,000$ as an approximation of the asymptotic size.

The deterministic mean \mathbf{m}_t of regression includes different combinations of drift, linear trend, seasonal dummies and seasonal drifts, since the inclusion of such regressors affects both the exact and the asymptotic distribution of the statistics. These combinations are shown in the first column of the tables.

The critical values of the statistic t_1 were simply based on 24,000 replications. We noticed that the asymptotic distribution of the ‘even’ t-statistics (t_{2k} , $k=1, 2, 3$, say t_{even}) are exactly the same and, on the other hand, the same statement holds for the ‘odd’ t-statistics (t_{2k+1} , say t_{odd}). This matter allowed us to stack all the simulations for each t-statistic class (Beaulieu y Miron, 1993) so we had 72,000 experimental replications to compute the t_{even} and the t_{odd} distributions. Finally, since regressors are asymptotically mutually orthogonal, the distribution of the F-statistic, say $F_{2k, 2k+1}$, asymptotically converges to the distribution given by $(t_{2k}^2 + t_{2k+1}^2)/2$, say $F_{even, odd}$. Therefore, we can stack again all the simulations and we had 144,000 items to compute the statistical distribution of $F_{even, odd}$. The Monte-Carlo simulation process was computed using the mathematical package MATLAB 5.3. in a PC with a Pentium III processor.

APPENDIX B: CRITICAL VALUES (CONTINUED)

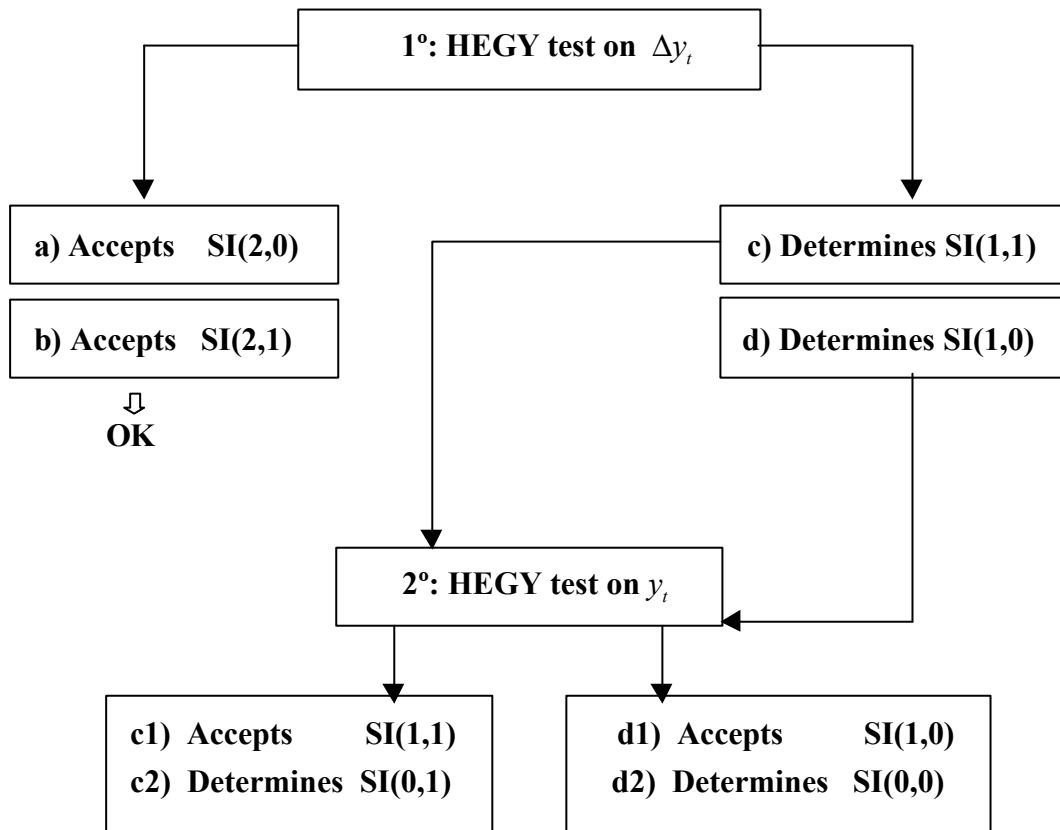
Auxiliary Regression	T	$t':p_1$				$t':p_{even}$				$F_{evenodd}$			
		0.01	0.025	0.05	0.10	0.01	0.025	0.05	0.10	0.90	0.95	0.97.5	0.99
No drift	240	-2.52	-2.16	-1.89	-1.57	-2.50	-2.17	-1.87	-1.54	2.32	3.01	3.66	4.63
No dummies	480	-2.58	-2.19	-1.91	-1.58	-2.54	-2.18	-1.89	-1.57	2.37	3.05	3.77	4.67
No trend	1000	-2.57	-2.21	-1.91	-1.60	-2.54	-2.20	-1.91	-1.58	2.39	3.08	3.78	4.70
Drift	240	-3.34	-3.04	-2.78	-2.49	-2.48	-2.13	-1.86	-1.53	2.30	2.98	3.65	4.54
Dummies	480	-3.43	-3.11	-2.84	-2.56	-2.50	-2.18	-1.89	-1.55	2.35	3.05	3.73	4.65
Trend	1000	-3.44	-3.12	-2.86	-2.57	-2.58	-2.23	-1.93	-1.58	2.38	3.08	3.79	4.73
Drift	240	-3.36	-3.04	-2.78	-2.47	-3.83	-3.52	-3.25	-2.95	5.25	6.25	7.18	8.33
No dummies	480	-3.39	-3.10	-2.81	-2.51	-3.86	-3.54	-3.30	-2.99	5.43	6.40	7.35	8.50
No trend	1000	-3.41	-3.10	-2.85	-2.56	-3.88	-3.58	-3.33	-3.03	5.54	6.53	7.48	8.63
Drift	240	-3.89	-3.60	-3.34	-3.06	-2.48	-2.16	-1.80	-1.55	2.30	2.97	3.67	4.52
No dummies	480	-3.98	-3.67	-3.41	-3.12	-2.51	-2.18	-1.89	-1.56	2.34	3.03	3.71	4.60
Trend	1000	-3.98	-3.67	-3.41	-3.12	-2.53	-2.20	-1.92	-1.58	2.40	3.10	3.81	4.70
Drift	240	-3.91	-3.57	-3.31	-3.03	-3.84	-3.52	-3.25	-2.95	5.24	6.23	7.16	8.32
Dummies	480	-3.93	-3.61	-3.35	-3.06	-3.86	-3.55	-3.29	-2.98	5.40	6.37	7.30	8.60
Trend	1000	-3.98	-3.67	-3.41	-3.12	-3.89	-3.58	-3.33	-3.03	5.56	6.59	7.54	8.69
No drift	240	-4.07	-3.76	-3.49	-3.19	-4.92	-4.59	-4.33	-4.03	8.88	10.12	11.29	12.81
Dummies	480	-4.03	-3.71	-3.46	-3.17	-4.81	-4.51	-4.25	-3.96	8.71	9.90	11.03	12.42
Seas. Trends.	1000	-3.98	-3.69	-3.42	-3.14	-4.72	-4.44	-4.19	-3.91	8.60	9.82	10.88	12.22

APPENDIX B. CRITICAL VALUES (CONTINUED)

Auxiliary Regression	T	$t' : p_{odd}$							
		0.01	0.025	0.05	0.10	0.90	0.95	0.97.5	0.99
No drift	240	-2.29	-1.93	-1.62	-1.25	1.26	1.62	1.94	2.31
No dummies	480	-2.31	-1.94	-1.62	-1.26	1.27	1.64	1.95	2.30
No trend	1000	-2.30	-1.95	-1.63	-1.29	1.28	1.63	1.95	2.30
Drift	240	-2.26	-1.90	-1.60	-1.24	1.27	1.63	1.96	2.33
No dummies	480	-2.32	-1.94	-1.63	-1.26	1.27	1.63	1.94	2.30
No trend	1000	-2.30	-1.95	-1.63	-1.27	1.27	1.63	1.94	2.30
Drift	240	-2.62	-2.21	-1.86	-1.45	1.48	1.89	2.24	2.63
Dummies	480	-2.67	-2.25	-1.90	-1.48	1.52	1.92	2.28	2.69
No trend	1000	-2.70	-2.29	-1.93	-1.52	1.54	1.95	2.29	2.70
Drift	240	-2.25	-1.89	-1.58	-1.22	1.26	1.62	1.93	2.32
No dummies	480	-2.31	-1.93	-1.62	-1.25	1.27	1.63	1.95	2.31
Trend	1000	-2.33	-1.96	-1.63	-1.26	1.27	1.63	1.95	2.34
Drift	240	-2.60	-2.21	-1.85	-1.45	1.45	1.85	2.21	2.63
Dummies	480	-2.66	-2.26	-1.91	-1.50	-1.49	1.89	2.24	2.65
Trend	1000	-2.70	-2.30	-1.93	-1.51	1.51	1.94	2.31	2.71
No drift	240	-2.65	-2.27	-1.90	-1.49	1.54	1.97	2.33	2.76
Dummies	480	-2.76	-2.34	-1.98	-1.55	1.58	2.01	2.37	2.81
Seas. dummies.	1000	-2.87	-2.45	-2.07	-1.62	1.63	2.08	2.44	2.85

APPENDIX C: TESTING SEQUENTIAL PROCEDURE

This appendix is devoted to showing the sequential test procedure when we check the integration order of the data generating process of a time series. Note that we define the time series y_t as seasonally integrated of orders d_0 and d_s , denoted as $SI(d_0, d_s)$, if the series $(1-B)^{d_0} S(B)^{d_s} y_t$ is stationary and invertible, where $S(B) = (1+B+B^2+\dots+B^{s-1})$. The HEGY test on the $SI(2,1)$ hypothesis applied to electricity log-demand series has power against lower order alternatives if we follow the procedure sketched below:



We should remark that the application of this procedure could evidence complex unit roots in some (but not all) seasonal frequencies. Thus the $S(B)$ filter is not really needed in order to ensure stationarity ($d_s = 0$), but the time series is not seasonal-stationary. Furthermore, the unit root test applied in the earlier step on the zero frequency will evidence either $d_0 = 2$ or $d_0 = 1$. So, following the methodological basis of this procedure, we should still test the stochastic trend hypothesis ($d_0 = 1$) against stationarity in the latter case, thereby applying the HEGY test on the level of the series. Therefore, the testing procedure applied is general enough to detect all relevant cases.

REFERENCES

- Banarjee, A.; Dolado, J.J.; Galbraith, J.W.; Hendry, D.F.; (1994) *“Cointegration, Error-Correction and the Econometric Analysis of Non-Stationarity Data”*, Oxford University Press.
- Beenstock, M.; Goldin, E.; Nabot, D.; (1999) *“The Demand for Electricity in Israel”*, Energy-Economics; 21(2), pp. 168-183.
- Beaulieu, J.J.; Miron, J.A.; (1993) *“Seasonal Unit Root in Aggregate U.S. Data”*, Journal of Econometrics, 55, pp. 305-328.
- Box, G.E.P; Jenkins, G.M.; (1970) *“Time Series Analysis: Forecasting and Control”*, San Francisco: Holden-Day.
- Bunn, D.; Farmer, E.D.; (1985) *“Comparative Models for Electrical Load Forecasting”*, New York: John Wiley.
- Cancelo, J.R.; Espasa, A.; (1996) *“Modelling and Forecasting Daily Series of Electricity Demand”*, Investigaciones Económicas, 20:3, pp.349-376.
- De Vany, A.S.; Walls, W.D.; (1999) *“Cointegration Analysis of Spot Electricity Prices: Insights on Transmission Efficiency in the Western US”*, Energy Economics, 21, pp. 435-448.
- Dickey, D.A.; Fuller, W.A; (1979) *“Distribution of the Estimators for Autoregressive Time Series with a Unit Root”*, Journal of the American Statistical Association 74, pp. 427-431.
- Dickey, D.A.; Pantula, S.G.; (1987) *“Determining the Order of Differencing in Autoregressive Processes”*, Journal of Business and Economic Statistics, 5, pp.455-461.
- Diebold, F.X.; Kilian, L.; (2000) *“Unit Roots are Useful for Selecting Forecasting Models”*, NBER Working Paper Series, PP-6928.
- Engle, R.; Granger, C.W.; (1987) *“Cointegration and Error Correction: Representation, Estimation and Testing”*, Econometrica, 55, pp. 251-276.

- Engle, R.; Granger, C.W.; Hallman, J.S.; (1989) “*Merging Short and Long-run forecasts: an application of seasonal cointegration to monthly electricity sales forecasting*”, Journal of Econometrics, 40:1, pp. 45-62.
- Johansen, S.; Schaumburg, E.; (1999) “*Likelihood Analysis of Seasonal Cointegration*”, Journal of Econometrics, 54, pp. 1-49.
- Ghysels, E.; Perron, P.; (1993) “*The effect of Seasonal Adjustment Filters of Tests for a Unit Root*”, Journal of Econometrics 55, pp. 57-98.
- Ghysels, E.; Lee, H.S.; Noh, J.; (1994) “*Testing for Unit Roots in Seasonal Time Series*”, Journal of Econometrics, 62, pp.415-442.
- Harvey, A.; Koopman, S.J; (1993) “ *Forecasting Hourly Electricity Demand Using Time-Varying Splines*”, Journal of the American Statistics Association, 88:424, pp. 1228-1236.
- Hylleberg, S.;Engle R.F.; Granger, C.W.J.; Yoo, B.S.; (1990) “*Seasonal Integration and Cointegration*”, Journal of Econometrics; 44, pp. 215-238.
- Lee, H.; (1992) “*Maximum Likelihood in Inference on Cointegration and Seasonal Cointegration*”, Journal of Econometrics 54, pp.1-47.
- López Milla, J.; (1999) “*La Liberalización del Sector Eléctrico Español. Una Reflexión a la Luz de la Experiencia de Inglaterra y Gales*”. Thesis dissertation, University of Alicante.
- Mastrángelo, S.; (2001) “*Legislación y Regulación del Mercado Eléctrico*”, in Introducción al Conocimiento del Mercado Eléctrico Mayorista, Technologic Institute of Buenos Aires.
- Ng, S.; Perron, P.; (1995) “*Unit Root Tests in ARMA Models with Data-dependent Methods for the Selection of the Truncation Lag*”, Journal of the American Statistical Association, 90, pp. 268-81
- O’Connor; M.J.; Kapoor, S.G.; (1984) “*Time series analysis of Building Electrical Load*”, in Time Series Analysis: Theory and Practice 5, Anderson O.D. (ed), North Holland.

Psaradakis, Z; (1997); *“Testing for Unit Roots in Time Series with Nearly Deterministic Seasonal Variation”*, *Econometric Reviews*, 16:4, pp. 421-439.

Rodriguez, J.; (2000) *“Constrained Nonparametric Regression Analysis of Load Curves”*, *Empirical Economics*; 25:2, pp. 229-246.

Wolak, F.A.; (1997) *“Market Design and Price Behaviour in Restructured Electricity Markets: An International Comparison”*, University of California Energy Institute, PWP-051.